This paper has been presented at ICSTCC 2012, Sinaia, Romania

Simple approaches to start-up anaerobic digestion systems for biogas production*

Mihaela Sbarciog, Jaime A. Moreno, and Alain Vande Wouwer

***Abstract*— The start-up of anaerobic digestion processes is a complex problem, which determines the entire evolution of the system. In spite of its importance, this issue has not received so far a significant attention. This paper presents two simple start-up strategies for an anaerobic digestion process, which have been developed based on the system dynamics. Both strategies consists of switching the dilution rate between minimum and maximum levels and then to an optimal level to drive the system towards a steady state characterized by maximum production of biogas. The main difference between the two policies consists in the type of measurement assumed: while for the first strategy it is supposed that the entire state of the system is available, for the second approach only the outflow rate of biogas is needed. Simulation results are presented to illustrate the principle of the two approaches.**

I. INTRODUCTION

Anaerobic digestion is a consolidated technology with remarkable advantages, such as low energy consumption, less sludge production compared to aerobic processes and direct production of biogas. Production of biogas provides a versatile carrier of renewable energy, as methane can be used for replacement of fossil fuels in both heat and power generation and as a vehicle fuel [1]. With more than 2200 high-rate reactors implemented worldwide, the total annual potential production of this renewable energy source is estimated at around 200 billions m^3 [2]. In Europe, the number of installed plants increased from 15 to 200 between 1995 and 2010 [3]. However, due to the process complexity, substantial expertise is required to properly operate such plants.

One of the most important applications in anaerobic digestion is the biomethanization of organic solid waste, mainly sewage sludge and municipal solid waste. This paper presents two simple start-up strategies for an anaerobic digestion process degrading organic solid wastes. The goal of both approaches is to drive the system from an arbitrary initial state to an optimal steady state, characterized by maximum production of biogas. These strategies rely on the analysis of system dynamics and consist of switching the manipulated variable between several levels. While the first strategy determines the switching instants by approximating the solution of a transient optimization problem, the switchings in the second approach take place when a decrease in the biogas production occurs.

*This paper presents research results of the Belgian Network DYSCO (Dynamical Systems, Control, and Optimization), funded by the Interuniversity Attraction Poles Programme, initiated by the Belgian State, Science Policy Office. The scientific responsibility rests with its author(s).

M. Sbarciog and A. Vande Wouwer are with the Automatic Control Laboratory, University of Mons, 31 Boulevard Dolez, 7000 Mons, Belgium MihaelaIuliana.Sbarciog@UMons.ac.be, Alain.VandeWouwer@UMons.ac.be

J.A. Moreno is with Coordinación Eléctrica y Computación, Instituto de Ingeniería, UNAM, 04510 Mexico D.F., Mexico JMorenoP@ii.unam.mx

II. SYSTEM DYNAMICS

Two-step reaction models are the most encountered ones in the analysis and control design of anaerobic digestion systems. The process model used in this paper [4] considers two reactions: hydrolysis and methanogenesis. The biological transformations are described by the following reaction network:

$$\begin{aligned} \xi_1 &\xrightarrow{r_1} a\xi_2 + \xi_3 \\ b\xi_2 &\xrightarrow{r_2} CH_4 + \xi_4 \end{aligned} \quad (1)$$

In the first reaction acidogens (ξ_3) transforms the particulate organic matter (ξ_1) into volatile fatty acids (ξ_2). In the second reaction, methanogens (ξ_4) consume the volatile fatty acids (ξ_2) and produce methane. $a, b > 0$ are the stoichiometric coefficients. The two reactions are characterized respectively by Contois and Haldane kinetics:

$$r_1(\xi) = \mu_{m_1} \frac{\xi_1}{K_x\xi_3 + \xi_1} \frac{K_{i_1}}{K_{i_1} + \xi_2} \cdot \xi_3 = \mu_1(\xi) \cdot \xi_3 \quad (2)$$

$$r_2(\xi) = \mu_{m_2} \frac{\xi_2}{K_s + \xi_2} \frac{K_{i_2}}{K_{i_2} + \xi_2} \cdot \xi_4 = \mu_2(\xi_2) \cdot \xi_4 \quad (3)$$

where μ_{m_1} is the maximum specific hydrolysis rate; K_x is the half saturation coefficient for the ratio ξ_1/ξ_3; K_{i_1} is the inhibition coefficient of the acidogens by volatile fatty acids; μ_{m_2} is the maximum specific methanogenesis rate; K_s is the half saturation coefficient for ξ_2; K_{i_2} is the inhibition coefficient of methanogens by volatile fatty acids.

For an ideal continuous stirred tank reactor operated at constant temperature, the system dynamics described by the reaction network (1) are given by the differential equations:

$$\dot{\xi}_1 = u(\xi_{in_1} - \xi_1) - r_1(\xi) \quad (4)$$

$$\dot{\xi}_2 = u(\xi_{in_2} - \xi_2) + ar_1(\xi) - br_2(\xi) \quad (5)$$

$$\dot{\xi}_3 = -u\xi_3 + r_1(\xi) \quad (6)$$

$$\dot{\xi}_4 = -u\xi_4 + r_2(\xi) \quad (7)$$

where u represents the dilution rate, ξ_{in_1} and ξ_{in_2} respectively represent the concentrations of the particulate organic matter and volatile fatty acids in the influent. The outflow rate of methane produced during the process is given by:

$$Q(\xi) = q \cdot \mu_2(\xi_2) \cdot \xi_4 \quad (8)$$

TABLE I

NUMERICAL VALUES OF MODEL PARAMETERS AND INPUTS

μ_{m_1}	6.8 d^{-1}	μ_{m_2}	1.19 d^{-1}
K_x	10.8 g/L	K_s	0.021 g/L
K_{i_1}	15 g/L	K_{i_2}	1.5 g/L
a	0.15	b	0.08
ξ_{in_1}	[0, 30] g/L	ξ_{in_2}	[0, 5] g/L
u	[0, 5] d^{-1}		

where q is the methane yield coefficient. Table I gives the numerical values of the parameters and input ranges used in this study.

By considering the state transformation: $x_1 = \xi_1 + \xi_3$, $x_2 = \xi_2 - a\xi_3 + b\xi_4$, $x_3 = \xi_3$, $x_4 = \xi_4$, with the positiveness constraints for the original states $S_x = \left\{x \in \mathbb{R}^4, x_1 - x_3 \geq 0, x_2 + ax_3 - bx_4 \geq 0, x_3 \geq 0, x_4 \geq 0\right\}$, an equivalent canonical representation of (4)-(7) can be obtained:

$$\dot{x}_1 = u\left(\xi_{in_1} - x_1\right) \quad (9)$$

$$\dot{x}_2 = u\left(\xi_{in_2} - x_2\right) \quad (10)$$

$$\dot{x}_3 = -ux_3 + r_1(\xi) \quad (11)$$

$$\dot{x}_4 = -ux_4 + r_2(\xi) \quad (12)$$

Based on this representation, it is obvious that all steady states of the system lie on the plane

$$\Delta = \left\{x \in \mathbb{R}^4, x_1 = \xi_1 + \xi_3 = \xi_{in_1}, x_2 = \xi_2 - a\xi_3 + b\xi_4 = \xi_{in_2}\right\}$$

and are the solutions of

$$x_1 = \xi_{in_1} \quad (13)$$

$$x_2 = \xi_{in_2} \quad (14)$$

$$[-u + \mu_1(\xi)] \cdot \xi_3 = 0 \quad (15)$$

$$[-u + \mu_2(\xi_2)] \cdot \xi_4 = 0 \quad (16)$$

In [5], the authors have shown that up to six physical steady states may occur in this system and have determined the necessary conditions for their occurrence. The number of physical steady states and their stability are determined by the dilution rate and concentration of substrates in the influent. There are four types of steady states, characterized by:

1) **Total wash out** ($\xi_3 = 0$, $\xi_4 = 0$): there exists one steady state of this type, denoted by ξ_A, which is always physical, independent of the magnitude of the dilution rate;
2) **Wash out of methanogenic bacteria** ($\xi_4 = 0$, $\mu_1(\xi) = u$): there may exist only one steady state of this type, denoted by ξ_B, where $\xi_{3,B}$ is the solution of

$$\mu_1(\xi) = u \quad (17)$$

for $\xi_1 = \xi_{in_1} - \xi_{3,B}$ and $\xi_2 = \xi_{in_2}$.
3) **Wash out of acidogenic bacteria** ($\xi_3 = 0$, $\mu_2(\xi_2) = u$): there may exist at most two steady states of this type, ξ_C and ξ_D, generically denoted by ξ_M, where $\xi_{2,C}$ and $\xi_{2,D}$ (with $\xi_{2,C} < \xi_{2,D}$) are the two solutions of

$$\mu_2(\xi_2) = u \quad (18)$$

4) **Coexistence of the two bacterial populations** ($\mu_1(\xi) = u$, $\mu_2(\xi_2) = u$): there may exist two steady states of this type, ξ_E and ξ_F, generically denoted by ξ_N. $\xi_{2,E}$ and $\xi_{2,F}$ (with $\xi_{2,E} < \xi_{2,F}$) are the two solutions of (18), while $\xi_{3,E}$ and $\xi_{3,F}$ are respectively the solutions of (17) for $\xi_1 = \xi_{in_1} - \xi_{3,E}$, $\xi_2 = \xi_{2,E}$ and $\xi_1 = \xi_{in_1} - \xi_{3,E}$, $\xi_2 = \xi_{2,E}$.

TABLE II

ANALYTICAL EXPRESSION OF SYSTEM EQUILIBRIA AND CONDITIONS FOR THEIR OCCURRENCE

Steady state	Conditions of occurrence
$\xi_A = \begin{bmatrix} \xi_{in_1} \\ \xi_{in_2} \\ 0 \\ 0 \end{bmatrix}$	n/a
$\xi_B = \begin{bmatrix} \xi_{in_1} - \xi_{3,B} \\ \xi_{in_2} + a\xi_{3,B} \\ \xi_{3,B} \\ 0 \end{bmatrix}$	$u \leq \mu_{m_1} K_{i_1} / (K_{i_1} + \xi_{in_2})$
$\xi_M = \begin{bmatrix} \xi_{in_1} \\ \xi_{2,M} \\ 0 \\ (\xi_{in_2} - \xi_{2,M})/b \end{bmatrix}$	$u \leq \max(\mu_2(\xi))$ $\xi_{2,M} \leq \xi_{in_2}$, $M = C, D$
$\xi_N = \begin{bmatrix} \xi_{in_1} - \xi_{3,N} \\ \xi_{2,N} \\ \xi_{3,N} \\ (\xi_{in_2} - \xi_{2,N} + a\xi_{3,N})/b \end{bmatrix}$	$u \leq \max(\mu_2(\xi))$ $u \leq \mu_{m_1} K_{i_1} / (K_{i_1} + \xi_{in_2})$ $\xi_{2,N} - a\xi_{3,N} \leq \xi_{in_2}$, $N = E, F$

Table II presents the analytical expression of the system equilibria as well as the conditions for their physical occurrence. Table II shows that, except for the total wash out state, which is independent of the input values, one or more conditions must be satisfied for a steady state to be physical. These conditions are graphically illustrated in Fig. 1 by continuous lines. They define in the space $\xi_{in_2} - u$ various regions, characterized either by a different number or type of steady states. The correspondence is given in Table III. Note that only the last condition for ξ_N involves ξ_{in_1}. In order to obtain a clear representation of the various regions, this condition has been evaluated for a specific value of ξ_{in_1} rather than an interval, and led to the curve separating regions 3 and 5 from regions 2, 4 and 6. This implies that the size of these regions will change depending on the value of ξ_{in_1}.

From a practical point of view, the only interesting equilibria are type N, characterized by the presence of both bacterial populations, which ensure the conversion of the substrates and the production of biogas. Among these two steady states, higher outflow rate of biogas is obtained in the equilibrium E than in F as ξ_E is characterized by a higher concentration of methanogenic bacteria. Moreover, ξ_E is always stable while ξ_F is always unstable. High efficiency (in terms of the amount of treated waste) of the anaerobic system and high biogas production are achieved by operating the process with high dilution rate such that

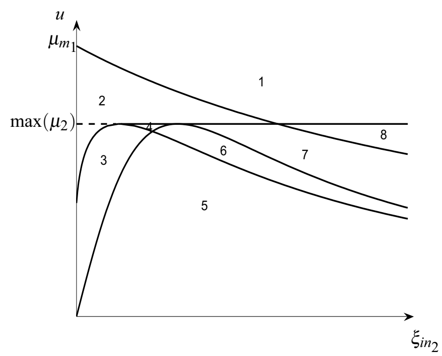

Figure 1. Regions corresponding to the occurrence of various steady states

TABLE III

CORRESPONDENCE BETWEEN THE REGIONS AND THE PHYSICAL STEADY STATES

Region	Physical steady states (s-stable, u-unstable)
1	ξ_A(s)
2	ξ_A(u), ξ_B(s)
3	ξ_A(u), ξ_B(u), ξ_E(s)
4	ξ_A(u), ξ_B(s), ξ_E(s), ξ_F(u)
5	ξ_A(u), ξ_B(u), ξ_C(u), ξ_E(s)
6	ξ_A(u), ξ_B(s), ξ_C(u), ξ_E(s), ξ_F(u)
7	ξ_A(u), ξ_B(s), ξ_C(u), ξ_D(u), ξ_E(s), ξ_F(u)
8	ξ_A(s), ξ_C(s), ξ_D(u)

the corresponding ξ_E is reached. However, ξ_E is the only stable equilibrium point of the system at low dilution rates; for higher dilution rates, ξ_B (methanogens wash out) and the operational point are both locally asymptotically stable. Hence for some initial conditions, which may represent a wide subset of the state space, the system will be irreversibly driven to the acidification point ξ_B, where volatile fatty acids are accumulating in the reactor as their conversion to biogas does not take place due to the absence of methanogenic bacteria. Thus, an efficient control strategy is needed to safely start-up the reactor and drive it towards a meaningful steady state in terms of high biogas production and efficient treatment of wastewater.

III. THE OPTIMAL SETPOINT

The optimal setpoint (ξ_s) is the steady state characterized by maximum outflow rate of methane. This steady state and the corresponding optimal dilution rate (u_s) are computed based on the system model. ξ_s is an equilibrium point of type E, having the analytical expression given in Table II, with $\xi_{2,s}$ the smaller solution of

$$\mu_2(\xi_2) = u_s \tag{19}$$

and $\xi_{3,s}$ the unique solution of

$$\mu_1(\xi)|_{\xi_1=\xi_{in_1}-\xi_{3,s},\xi_2=\xi_{2,s}} = u_s \tag{20}$$

Thus, in the optimal equilibrium point ξ_s the flow rate of methane is given by

$$Q(\xi_s) = q\mu_2(\xi_{2,s})\frac{1}{b}(\xi_{in_2} - \xi_{2,s} + a\xi_{3,s}) \tag{21}$$

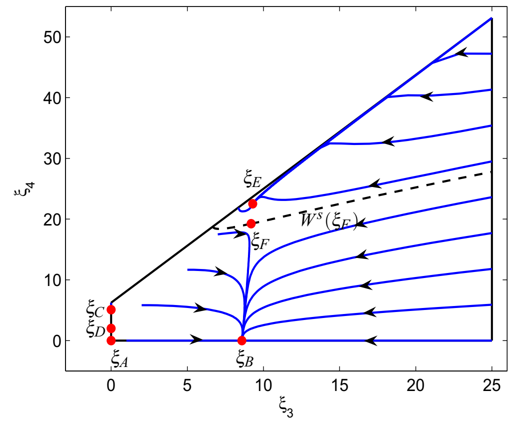

Figure 2. The phase portrait on the plane Δ of the system operated with the dilution u_s

Using (19) and (20) to express $\xi_{3,s}$ in function of $\xi_{2,s}$ and introducing this expression in (21), one obtains $Q(\xi_s) = Q(\xi_{2,s})$. Then,

$$\frac{dQ(\xi_{2,s})}{d\xi_{2,s}} = 0 \tag{22}$$

together with (19) and (20) provide sufficient conditions to fully determine the optimal setpoint ξ_s and the optimal dilution rate u_s.

For fixed values of the inlet concentrations $\xi_{in_1} = 25$ g/L, $\xi_{in_2} = 0.5$ g/L, which will be further used in all simulations, the optimal steady state and the corresponding dilution rate are

$$\xi_s = \begin{bmatrix} 15.704 & 0.093 & 9.296 & 22.518 \end{bmatrix}'$$
$$u_s = 0.914\mathrm{d}^{-1}$$

Fig. 2 shows the phase portrait of the system operated with the optimal dilution rate on the plane Δ, illustrating the reduced size of the attraction region of the optimal steady state, which motivates the need of using efficient start-up strategies for safely driving the system in the neighborhood of ξ_s.

IV. THE START-UP POLICIES

The classic start-up methodology in anaerobic digestion systems is to operate the process with a low, constant dilution rate in order to avoid the wash out of bacteria, especially the methanogens. This allows the system to build up a considerable amount of biomass and to reach a steady state of type E. Then, a controller is introduced in the loop for achieving some desired objectives. When successful, this start-up takes a long period of time, the amount of treated waste is small due to the low dilution rate, while the amount of produced biogas is insignificant. The start-up strategies presented here have as main goal to bring the system to the neighborhood of the optimal equilibrium point ξ_s (characterized by maximum biogas production) while ensuring an increase in the outflow rate of biogas. The control law for both strategies consists of switching the dilution rate between minimum and maximum levels (u_{min} and u_{max}) until the system enters the neighborhood of the optimal equilibrium ξ_s (further called target set and denoted by S). Then the dilution rate is changed to u_s, the optimal dilution rate, which allows the system to settle down in the optimal setpoint.

The boundary of the target set S is defined as

$$\sum_{i=1}^{4} \frac{(\xi_i - \xi_{i,s})^2}{y_i} = 1 \tag{23}$$

where y_i, with $i = 1 \ldots 4$ must be chosen such that $S \subset \Omega(\xi_s)$ (where $\Omega(\xi_s)$ denotes the attraction region of ξ_s) and $\xi_E^{u_{max}} \in S$ (where $\xi_E^{u_{max}}$ denotes the equilibrium point of type E of the system operated with the dilution u_{max}). Here the following values of the parameters have been selected: $y_1 = 0.5$, $y_2 = 0.1$, $y_3 = 0.7$, $y_4 = 1.5$.

A. An optimizing start-up strategy

Since one of the main interests is to increase the biogas outflow rate, the start-up strategy may be formulated as a free final time optimal control problem of the form: *Find the dilution rate $u(t) \in [u_{min}, u_{max}]$, which drives the system from an initial state at time $t = 0$ to the target set S in finite time, while minimizing a cost index of the form*

$$J(u) = \int_0^{t_f} (\alpha \cdot u - Q(\xi))\, dt \tag{24}$$

where t_f represents the final time of the control interval and α is a weighting coefficient.

This start-up policy has been proposed in [6], [7] for an anaerobic digestion model in which the acidogenesis is described by Monod kinetics and the methanogenesis is described by Haldane kinetics. The canonical representation and maximum principle of Pontryagin are used to determine the structure of the control. It is shown that singular intervals cannot occur, thus the optimal control is bang-bang. However, the determination of the exact control law requires the solution of a problem with split boundary conditions, which constitutes a difficult numerical problem. In order to avoid solving this complicated numerical problem, a suboptimal control law with only one switching from u_{min} to u_{max} is employed, where the switching is chosen to take place on $\partial\Omega(\xi_E^{u_{max}}) = W^s(\xi_F^{u_{max}})$, the system stability boundary delimiting the region of attraction of $\xi_E^{u_{max}}$ (the type E equilibrium point of the system operated with u_{max}). This can be accurately estimated using a trajectory reversing technique [8], [9].

Although the existence of singular intervals in the optimal solution cannot be a priori excluded for model (9)-(12) due to the more complex kinetics of the hydrolysis reaction, the bang-bang start-up policy can be successfully applied to the anaerobic digestion of solid organic waste because the switching surface is chosen based on the system characteristics. No extensive simulations are employed to determine the surface for specific kinetics structure and numerical values of the parameters. Even if the kinetics of the hydrolysis reaction are different from the ones of the acidogenesis reaction, the structure of the phase portrait is similar. The start-up policy is summarized as follows:

- operate the system with minimum dilution rate u_{min} until the system trajectory reaches the stability boundary $W^s(\xi_F^{u_{max}})$;
- operate the system with the maximum dilution rate u_{max} until the target set S is reached;
- operate the system with the optimal dilution rate u_s.

Thus this start-up strategy consists of continuously monitoring the system states and checking if the stability boundary or the target set have been reached. Fig. 3 presents the outflow rate of biogas and the control law along a system trajectory, while Fig. 4 illustrates the phase portrait on the plane Δ of the system controlled with the optimizing start-up policy.

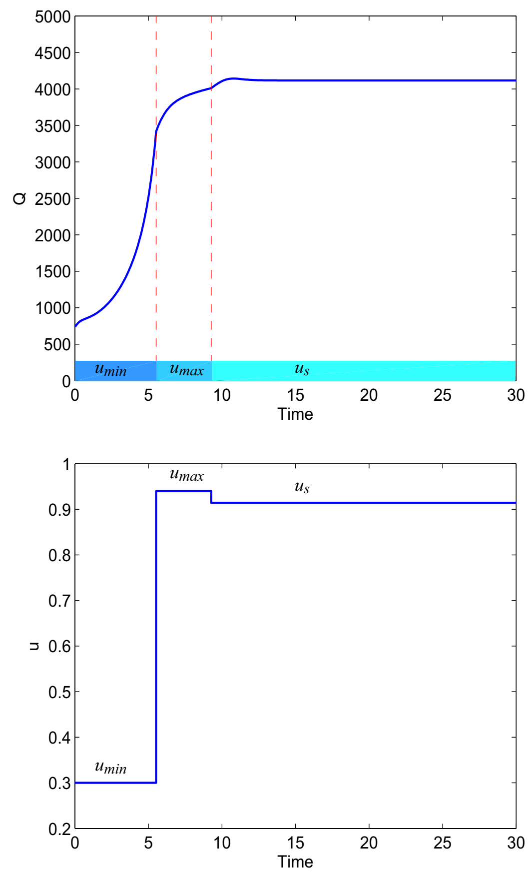

Figure 3. The biogas outflow rate and the control along a system trajectory controlled with the optimizing policy ($\xi(0) = [24.5 \;\; 24.83 \;\; 1 \;\; 4]'$)

B. A biogas-based switching policy

As one of the main goals is increasing the outflow rate of biogas, a natural approach of starting-up the anaerobic digestion process is by using a biogas-based switching policy, which consists of switching the dilution rate levels when

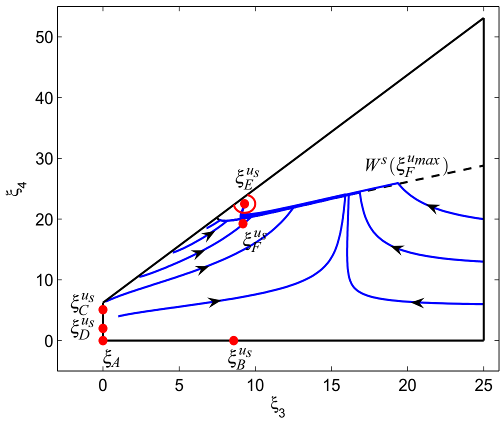

Figure 4. The phase portrait on the plane Δ of the system controlled with the optimizing policy

biogas production decreases [10]. In this case however, it is difficult to prove that only one switch from minimum to maximum dilution rate will bring the system to the target set from any possible initial state. This is due to the fact that the outflow rate of biogas is a nonlinear function of the system states, with several local maxima. Switching the dilution rate to the maximum value at one of the local maxima, without the possibility to switch back to the minimum level, may drive the system irreversibly to the acidification point $\xi_B^{u_{max}}$, where the methanogens are washed out. Therefore the switching policy is stated more generally, such that multiple switchings (if necessary) are allowed, occurring when the outflow rate of biogas is decreasing:

- the operation starts with u_{min};
- a switch from u_{min} to u_{max} occurs when the biogas decreases by ΔQ_1 (this avoids the unnecessary switching for small variations in the outflow rate of biogas);
- a switch from u_{max} to u_{min} occurs when the biogas decreases by ΔQ_2 (this avoids high frequency switchings occurring especially at high acidogens and methanogens concentrations due to the low levels of substrates in the system).

Fig. 5 presents the outflow rate of biogas and the control law along a system trajectory, while Fig. 6 shows the phase portrait on the plane Δ of the system controlled with the biogas-based switching policy.

V. Discussion and Conclusions

An important feature of the two start-up strategies is their ability of safely driving the system towards the optimal setpoint, enlarging in this way its region of attraction. Here, the optimal setpoint can be reached from any system initial state characterized by the presence of both bacteria type, by using any of the proposed start-up strategies. In fact, the size of the attraction region of ξ_s in the controlled case (denoted by $\Omega^c(\xi_s)$) is related to the minimum dilution rate level u_{min}, as $\Omega^c(\xi_s)$ will always equal $\Omega(\xi_E^{u_{min}})$. When u_{min}

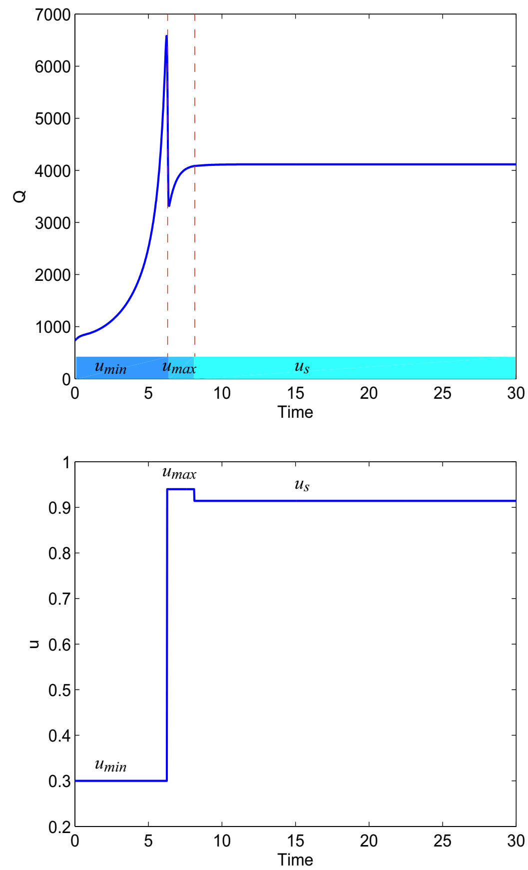

Figure 5. The biogas outflow rate and the control along a system trajectory controlled with the biogas-based switching policy ($\xi(0) = [24.5 \;\; 24.83 \;\; 1 \;\; 4]'$)

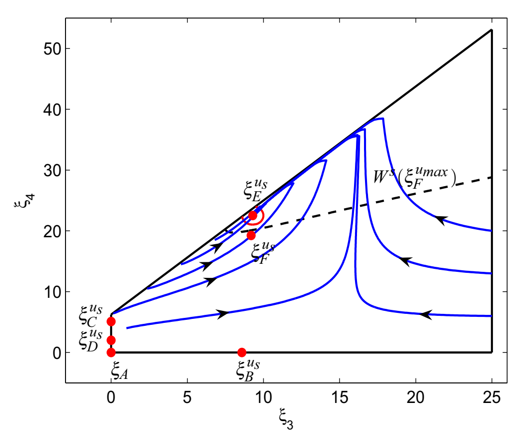

Figure 6. The phase portrait on the plane Δ of the system controlled with the biogas-based switching policy

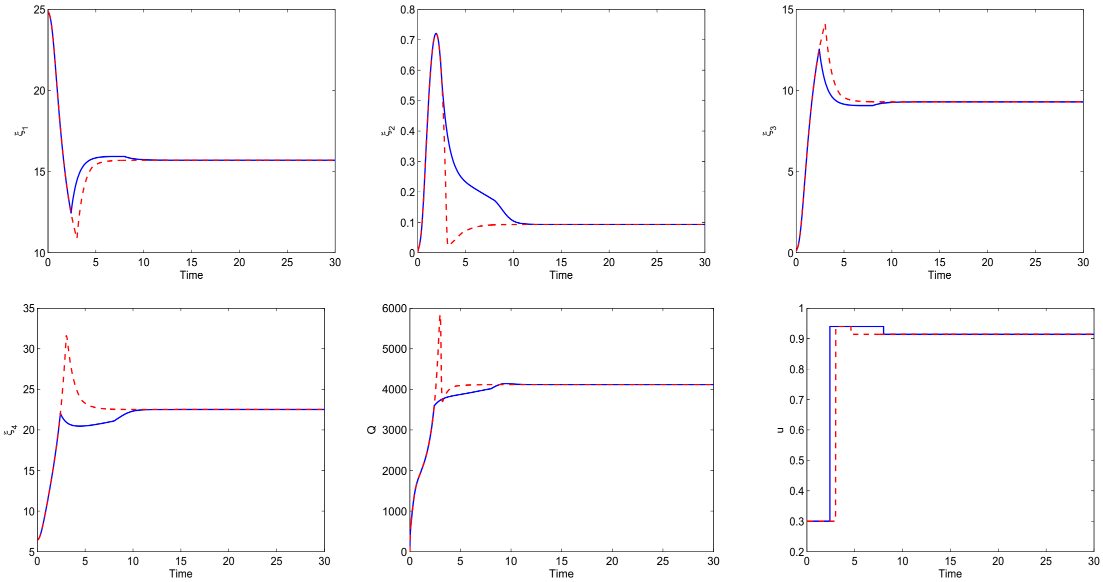

Figure 7. The system states, the outflow rate of biogas and the control laws: the optimizing strategy - continuous line, the biogas-based switching strategy - dashed line

is selected such that the pair (ξ_{in_2}, u_{min}) belongs to either region 3 or region 5 (see Fig. 1 and Table III), then $\Omega(\xi_E^{u_{min}})$ equals the interior of the state space (any state with $\xi_3 \neq 0$, $\xi_4 \neq 0$). If the pair (ξ_{in_2}, u_{min}) belongs to either region 4, 6 or 7, then $\Omega(\xi_E^{u_{min}})$ reduces its size, but it will be larger than $\Omega(\xi_s)$ as $u_{min} < u_s$. Thus, $\Omega^c(\xi_s)$ is always larger than $\Omega(\xi_s)$. However, particular care must be taken when selecting the system initial state, as in the last case wash out of methanogenic bacteria may occur also during the operation with the minimum dilution rate.

Fig. 7 presents the states, outflow rate of biogas and the control law for both start-up policies. Noticeably, the switching instants between the dilution rate levels are quite different. The evolution of the outflow rate of biogas suggests that the optimal steady state is reached faster in the case of the biogas-based switching strategy.

Both strategies are characterized by simplicity, consisting of switching between constant, predefined control levels. Their design exploits the intrinsic system characteristics. The most important difference between the two start-up policies lies in the difficulty of the real-life implementation. While the optimizing policy requires the knowledge of the full system state to determine whether or not the system stability boundary is reached, the biogas-based switching policy needs only the measurement of the outflow rate of biogas. One of the great challenges of modelling and controlling anaerobic digestion systems is the reduced number of available measurements. Hence the assumption that the full system state is measurable is highly optimistic. Although difficult, it is not impossible to implement the optimizing strategy in practice. On the other hand, the outflow rate of biogas can be easily measured in anaerobic digestion systems. This makes the application of the biogas-based switching strategy in real-life plants straightforward.

References

[1] P. Weiland, Biogas production: current state and perspectives, Appl Microbiol Biotechnol, 85, 849-860, 2010.

[2] L. Appels, J. Baeyens, J. Degreve, R. Dewil, Principles and potential of the anaerobic digestion of waste-activated sludge, Progress in Energy and Combustion Science, 34, 755-781, 2008.

[3] A. Donoso-Bravo, J. Mailier, C. Martin, J. Rodriguez, C. Aceves-Lara, A. Vande Wouwer, Model selection, identification and validation in anaerobic digestion : a review, Water Research, 45, 5347-5364, 2011.

[4] V.A. Vavilin, I. Angelidaki, Anaerobic degradation of solid material: Importance of initiation centers for methanogenesis, mixing intensity and 2D distributed model, Biotechnology and Bioengineering, 89, 113-122, 2005.

[5] M. Sbarciog, A. Donoso-Bravo, A. Vande Wouwer, Evaluation of steady state multiplicity for the anaerobic degradation of solid organic waste, Computer Aided Chemical Engineering, 29, 548-551, 2011.

[6] M. Sbarciog, M. Loccufier, A. Vande Wouwer, On the optimization of biogas production in anaerobic digestion systems, IFAC World Congress, Italy 2011.

[7] M. Sbarciog, M. Loccufier, A. Vande Wouwer, An optimizing start-up strategy for a bio-methanator, Bioprocess Biosyst Eng, 35, 565-578, 2012.

[8] M. Sbarciog, M. Loccufier, E. Noldus, The computation of stability boundaries in state space for a class of biochemical engineering systems, J Computational and Applied Mathematics, 215, 557-567, 2008.

[9] M. Sbarciog, M. Loccufier, E. Noldus, The estimation of stability boundaries for an anaerobic digestion system, Proceedings of the 11th International Symposium on Computer Applications in Biotechnology, 359-364, 2010.

[10] M. Sbarciog, J.A. Moreno, A. Vande Wouwer, A biogas-based switching control policy for anaerobic digestion systems, Preprints of the 8th IFAC Symposium on Advanced Control of Chemical Processes, 603-608, 2012.