This paper has been presented at ICA 2013, Narbonne, France

State and Unknown Input Estimation for an Anaerobic Digestion System

M. Sbarciog*, J.A. Moreno, E. Guzman**, A. Vande Wouwer***

*UMONS, Automatic Control Laboratory, 31 Boulevard Dolez, 7000 Mons, Belgium (e-mail: {MihaelaIuliana.Sbarciog, Alain.Vandewouwer}@umons.ac.be)
**UNAM, Coordinacíon Eléctrica y Computación, Instituto de Ingeniería, 04510 Mexico D.F., Mexico (e-mail:JMorenoP@ii.unam.mx)

Keywords: waste treatment; anaerobic digestion; estimation

Abstract: This paper presents the estimation of the unknown states and inputs of an anaerobic digestion system characterized by a two-step reaction model. The estimation is based on the measurement of the two substrate concentrations and of the outflow rate of biogas and relies on the use of three observers. The first is a generalized super-twisting observer, which estimates a linear combination of the two input concentrations. The second is an asymptotic observer, which provides one of the two biomass concentrations, whereas the third is a super-twisting observer for one of the input concentrations and the second biomass concentration.

INTRODUCTION

The anaerobic treatment of waste or wastewater has become lately very popular due to lower energy requirement, less biomass to be disposed and production of biogas (a mixture of carbon dioxide and methane), which can subsequently be used as a source of renewable energy. The involved biotransformations lead to a complex process, sensitive to input variations and requiring tight optimization and control. However, the implementation of advanced control techniques, able to ensure the process performance is hampered by the lack of measurements, which is usual in anaerobic digestion systems. Therefore the missing information needs to be inferred by means of unknown state/input estimation techniques from the available measurements (usually substrates concentration and outflow rate of biogas). Obviously, the accuracy of the estimation influences the performance of the control.

Several observer structures have been proposed for anaerobic digestion systems, among which the asymptotic observer and the interval observer are the most popular ones [1-3]. The asymptotic observers require the knowledge of the system inputs to provide accurate estimation of the states, while the interval observers do not provide a value but a range of variation for the estimated variable. In this paper we propose the use of a generalized super-twisting algorithm [4] for a two-step anaerobic digestion model to estimate in finite time the exact value of the variable of interest. The goal is to simultaneously estimate the concentrations of the two biomasses and the two nutrients in the influent assuming that (i) measurements of the two substrates and of biogas outflow rate are available and (ii) the structure of the model and its parameters are known. The distinguishing properties of the discontinuous generalized super-twisting observer compared to other smooth observers are convergence in finite time and insensitivity to time-varying but bounded perturbations [4]. These properties are crucial for achieving the proposed goal. To this end, three observers are implemented: two of them are based on the generalized super-twisting algorithm, the third one is an asymptotic observer. Simulation results are presented to illustrate the accuracy of the estimation.

SYSTEM DESCRIPTION

The anaerobic digestion model used in this paper is given by the following reaction network:

$$as_1 \rightarrow cs_2 + x_1; \qquad ds_2 \rightarrow CH_4 + x_2 \tag{1}$$

where the acidogenic bacteria x_1 grow on the organic substrate s_1 and produce volatile fatty acids s_2. Subsequently, the methanogenic bacteria x_2 use the volatile fatty acids as substrate for growth and produce methane. This is one of the most popular models used for the analysis and control of the anaerobic digestion of wastewater, as it is able to describe quite accurately the complex system dynamics with only two reactions.

For an ideal continuous stirred tank reactor, operated at constant temperature, the system dynamics described by the reaction network (1) are given by the following differential equations:

$$\begin{aligned} \dot{s}_1 &= u\left(s_{in_1} - s_1\right) - a\mu_1\left(s_1\right)x_1 \\ \dot{s}_2 &= u\left(s_{in_2} - s_2\right) + c\mu_1\left(s_1\right)x_1 - d\mu_2\left(s_2\right)x_2 \\ \dot{x}_1 &= -ux_1 + \mu_1\left(s_1\right)x_1 \\ \dot{x}_2 &= -ux_2 + \mu_2\left(s_2\right)x_2 \end{aligned} \tag{2}$$

where the outflow rate of methane gas reads:

$$Q\left(x_2, s_2\right) = q \cdot \mu_2\left(s_2\right) \cdot x_2 \tag{3}$$

with u - the dilution rate; s_{in_1}, s_{in_2} - the concentrations of organic substrate and of volatile fatty acids in the influent; $a,c,d > 0$ - the stoichiometric coefficients; $q > 0$ - the yield for the methane production. The growth functions are respectively of Monod and Haldane type:

$$\mu_1\left(s_1\right) = \mu_{m_1}\frac{s_1}{K_{S_1} + s_1}; \qquad \mu_2\left(s_2\right) = \mu_{m_2}\frac{s_2}{K_{S_2} + s_2 + s_2^2 / K_{I_2}} \tag{4}$$

Assuming that the model is known, but that the growth rate $\mu_2\left(s_2\right)$ may be uncertain in its form and/or its parameters, the problem consists of estimating the unmeasurable state variables x_1, x_2 and the unknown inputs s_{in_1}, s_{in_2} from the measured variables $\left[y_1 \quad y_2 \quad y_3\right]' = \left[s_1 \quad s_2 \quad Q\right]'$. The unknown input s_{in_1} is assumed to be constant (or slowly varying) and s_{in_2} can be arbitrarily time varying, with a bounded derivative.

OBSERVER STRUCTURE

Three observers are designed to estimate the non-measurable states and the unknown inputs. The first observer ensures a finite-time estimation of the unknown inputs combination $z_{in} = c/a \cdot s_{in_1} + s_{in_2}$, the second one asymptotically estimates the concentration of methanogenic bacteria x_2, while the last one estimates in finite-time the concentrations of acidogenic bacteria x_1 and of the organic substrate in the influent s_{in_1}. Due to space limitation only the structures of the three observers are given below:

1. By defining $z = c/a \cdot s_1 + s_2$, which is known since it is a combination of the measured variables, the observer O_1 can be constructed to estimate z_{in} in finite-time:

$$O_1 : \begin{cases} \dot{\hat{z}} = -L\, k_1^1\, \Phi_1^1(e_1) - uz - d/q\, y_3 + u\hat{z}_{in} \\ \dot{\hat{z}}_{in} = -L^2\, k_2^1\, u \Phi_2^1(e_1) \end{cases}; \qquad k_1^1, k_2^1 > 0 \tag{5}$$

$$e_1 = \hat{z} - z$$

where $\Phi_1^1(e_1) = \gamma_1^1 |e_1|^{\frac{1}{2}}\, sign(e_1) + \gamma_2^1 e_1$ (6)

$$\Phi_2^1(e_1) = \frac{(\gamma_1^1)^2}{2}\, sign(e_1) + \frac{3}{2}\gamma_1^1\gamma_2^1 |e_1|^{\frac{1}{2}}\, sign(e_1) + (\gamma_2^1)^2 e_1$$

γ_1^1 and γ_2^1 are non-negative constants to be designed.

2. The observer O_2 estimates asymptotically the value of x_2 whenever u is persistently exciting:

$$O_2 : \quad \dot{\hat{x}}_2 = -u\hat{x}_2 + 1/q \cdot y_3 \tag{7}$$

3. The observer O_3 estimates in finite-time x_1 and s_{in_1} under the assumption that s_{in_1} is constant or slowly varying such that $\dot{s}_{in_1} = 0$:

$$O_3 : \begin{cases} \dot{x} = -[u - \mu_1(y_1)]x, \quad x(0) = 1 \\ \dot{\hat{s}}_1 = -k_1^3 \Phi_1^3(e_3) - uy_1 - a\mu_1(y_1)\hat{x}_1 + u\hat{s}_{in_1} \\ \dot{\hat{x}}_1 = k_2^3 a \mu_1(y_1) x^2 \Phi_2^3(e_3) - [u - \mu_1(y_1)]\hat{x}_1 \\ \dot{\hat{s}}_{in_1} = -k_3^3 u \Phi_2^3(e_3) \end{cases}; \qquad k_1^3, k_2^3, k_3^3 > 0 \tag{8}$$

where $e_3 = \hat{s}_1 - s_1$ and Φ_1^3, Φ_2^3 have the same structure as in (6).

SIMULATION RESULTS AND CONCLUSIONS

Figure 4.1 shows the input signal and the estimations provided by all observers. An uncertainty of 20% on the initial conditions of the observers is assumed. In this simulation test, the influent concentration of the organic substrate s_{in_1} varies stepwise (from 40g/L to 60g/L and then to 30g/L), while the influent concentration of volatile fatty acids s_{in_2} has a periodic variation around a nominal value (175mmol/L). The parameters of the generalized super-twisting observers are set to $\gamma_1^1 = 1$, $\gamma_2^1 = 5$, $k_1^1 = 2.2$, $k_2^1 = 1.5$ and $L = 10$, $\gamma_1^3 = 1$, $\gamma_2^3 = 5$, $k_1^3 = 5$, $k_2^3 = 3.2e-3$, $k_3^3 = 3e3$.

A fast and accurate estimation of the variables of interest is achieved. Further developments involve the study of the robustness with respect to measurement noise and parametric uncertainty.

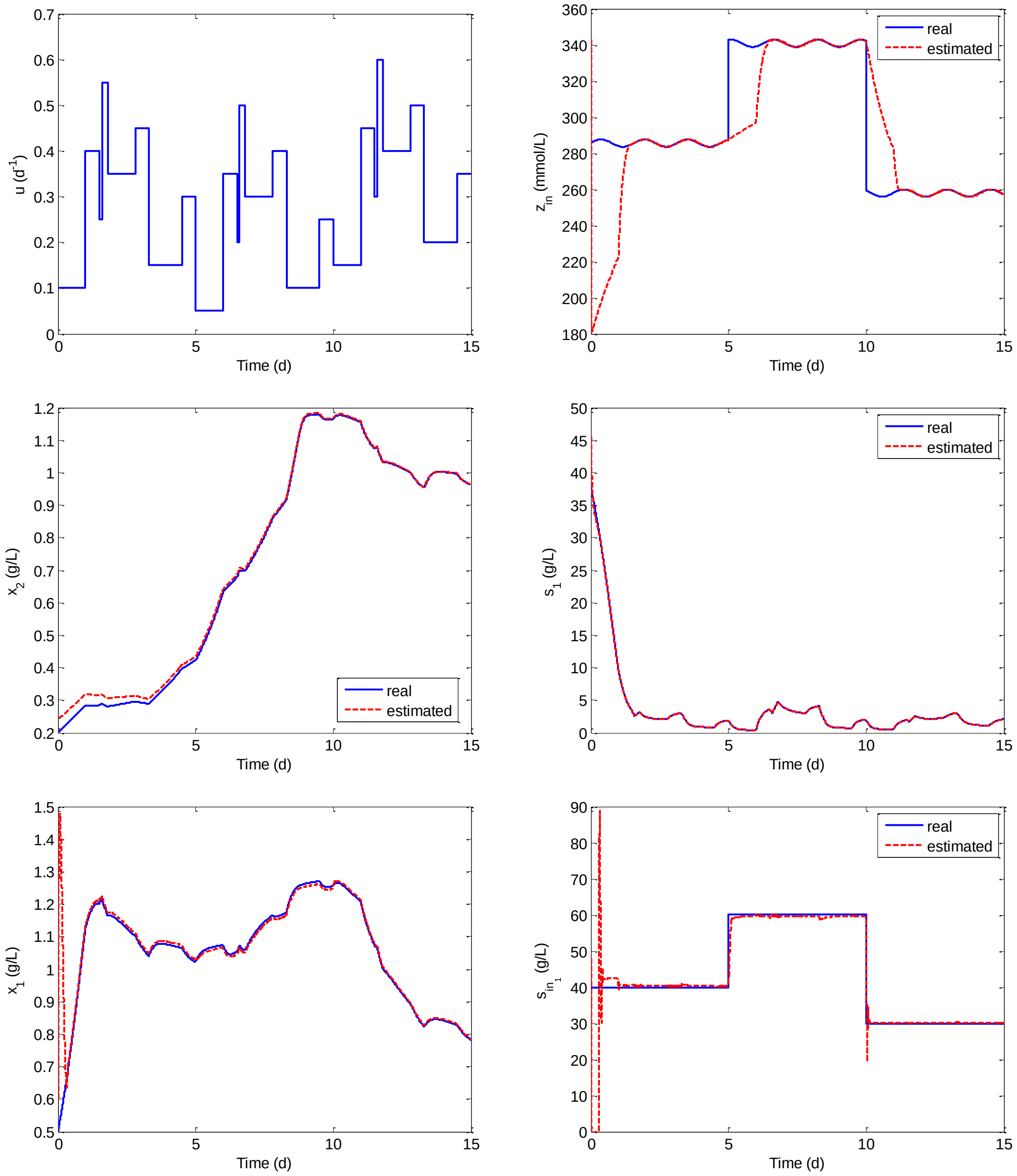

Figure 4.1. Estimations of the states and input concentrations: real variables (continuous line), estimated variables (dashed line)

REFERENCES

[1] Bastin G. & Dochain D. 1990 *On-line Estimation and Adaptive Control of Bioreactors*. Elsevier, Amsterdam.

[2] Theilliol D., Ponsart J-C., Harmand J., Join C. & Gras P. 2003 On-line estimation of unmeasured inputs for anaerobic wastewater treatment processes. *Control Engineering Practice* **11**, 1007-1019.

[3] Alcaraz-Gonzalez V., Harmand J., Rapaport A., Steyer J.P., Gonzalez-Alvarez V., Pelayo-Ortiz C. 2002 Software sensors for highly uncertain WWTPs:a new approach based on interval observers. *Water Research* **36**, 2515–2524.

[4] Moreno, J.A. 2011 Lyapunov Approach for Analysis and Design of Second Order Sliding Mode Algorithms. In: Fridman, L.; Moreno, J.; Iriarte, R. (Ed.) *Sliding Modes after the first decade of the 21st Century*. Springer. LNCIS Vol. 412, 113 -- 150.