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A Biogas-Based Switching Control Policy for Anaerobic Digestion Systems ⋆

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Abstract: This paper presents a simple switching control policy for driving an anaerobic digestion system from an arbitrary initial state to an optimal setpoint, characterized by the maximum biogas production. A high wastewater treatment efficiency is achieved and a fast convergence to the optimal setpoint is obtained. The main features of this control strategy are its simplicity and ease of implementation, as it is based on the measurement of the biogas outflow rate, which is commonly available in anaerobic digestion systems.

Keywords: biotechnology, waste treatment, nonlinear systems, system analysis, optimality, bang-bang control

1. INTRODUCTION

Anaerobic digestion is one of the most encountered process for the biological treatment of wastewater, as it not only removes the pollutants but also allows the production of biogas. Compared to the aerobic treatment, the anaerobic digestion provides several advantages among which the higher energy production and the substantially lower sludge production are the most important ones. In terms of process stability, anaerobic digestion still lags behind aerobic biological treatment or physico-chemical processes. Substantial expertise is required to operate such a process properly. From a biological point of view, the main cause of the anaerobic digestion failure is the imbalance between the acid forming bacteria and the methane forming bacteria.

Many control strategies for anaerobic digestion processes have been proposed in the literature, which aim to either regulate the organic pollution level or to optimize the production of the methane gas. The most popular ones are robust output feedback control (Mailleret et al., 2003; Antonelli et al., 2003; Méndez-Acosta et al., 2010) and adaptive control (Mailleret et al., 2004; Marcos et al., 2004; Dimitrova and Krastanov, 2009). Steyer et al. (2006) have reviewed a number of control strategies for anaerobic digestion systems and have concluded that neither the classical nor the advanced control methods have succeeded in overcoming all the difficulties which arise in the efficient operation of these processes. Therefore, more efficient and less complex control laws may be derived by exploiting the insight gained from a thorough analysis of system dynamics. This provides useful guidance for process operation and control (e.g. the methodology of detecting hazardous working modes developed by Hess and Bernard (2008, 2009)).

This paper presents a simple methodology for starting-up anaerobic digestion systems. The control law consists of switching the dilution rate between minimum and maximum levels such that the system is driven in the neighbourhood of an optimal setpoint. The optimal setpoint is the system equilibrium point characterized by the maximum biogas outflow rate, and it is computed by solving a steady state optimization problem. Once in the neighbourhood of the optimal setpoint, the dilution rate is set to the optimal value such that the system reaches the optimal state. A similar start-up strategy has been recently proposed by Sbarciog et al. (2011, 2012) and validated also in the case of multispecies anaerobic digestion systems (Sbarciog and Vande Wouwer, 2012). Although having the same goal (driving the system to the optimal setpoint), there exists a fundamental difference between the two strategies in the type of the assumed measurement.

2. PROCESS DESCRIPTION

Throughout this paper, an anaerobic digestion model is considered, in which the biological transformations are given by the following reaction network:

$$\begin{aligned} a\xi_1 &\to c\xi_2 + \xi_3 \\ d\xi_2 &\to CH_4 + \xi_4 \end{aligned} \qquad (1)$$

In the first reaction, the acidogenic bacteria ξ_3 grow on the organic substrate ξ_1 and produce volatile fatty acids ξ_2. In

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the second reaction, the methanogenic bacteria ξ_4 use the volatile fatty acids as substrate for growth and produce methane. When operating the anaerobic digestion process, a balance between the acidogenesis and methanogenesis must be maintained. This is one of the most popular models used for the analysis and control of the anaerobic digestion of wastewater, as it is able to describe quite accurately the complex system dynamics with only two reactions.

For an ideal continuous stirred tank reactor, operated at constant temperature, the system dynamics described by the reaction network (1) are given by the following differential equations:

$$\dot{\xi}_1 = u(\xi_{in_1} - \xi_1) - a\mu_1(\xi_1)\xi_3 \tag{2}$$

$$\dot{\xi}_2 = u(\xi_{in_2} - \xi_2) + c\mu_1(\xi_1)\xi_3 - d\mu_2(\xi_2)\xi_4 \tag{3}$$

$$\dot{\xi}_3 = -u\xi_3 + \mu_1(\xi_1)\xi_3 \tag{4}$$

$$\dot{\xi}_4 = -u\xi_4 + \mu_2(\xi_2)\xi_4 \tag{5}$$

while the outflow rate of methane gas reads:

$$Q(\xi) = q\mu_2(\xi_2)\xi_4 \tag{6}$$

In equations (2)-(6), u represents the dilution rate and ξ_{in_1}, ξ_{in_2} respectively represent the concentrations of organic substrate and of volatile fatty acids in the influent. $a, c, d > 0$ are the stoichiometric coefficients. $q > 0$ is the yield for the methane production. $\xi = [\xi_1 \;\; \xi_2 \;\; \xi_3 \;\; \xi_4]' \in \mathbb{R}^{+^4}$ denotes the state vector. The growth functions are respectively of Monod and Haldane type

$$\mu_1(\xi_1) = \mu_{m_1} \frac{\xi_1}{K_{s_1} + \xi_1} \tag{7}$$

$$\mu_2(\xi_2) = \mu_{m_2} \frac{\xi_2}{K_{s_2} + \xi_2 + \frac{\xi_2^2}{K_{i_2}}} \tag{8}$$

Table 1 gives the numerical values/ranges for the anaerobic digestion model parameters and input variables, used for the simulation results.

Table 1. Numerical values/ranges of the anaerobic digestion model parameters and input variables (as in Bernard et al. (2001))

a	42.14		K_{s_1}	7.1	g/l
c	116.5	mmol/g	K_{s_2}	9.28	mmol/l
d	268	mmol/g	K_{i_2}	256	mmol/l
q	453	mmol/g	ξ_{in_1}	40	g/l
μ_{m_1}	1.2	day^{-1}	ξ_{in_2}	175	mmol/l
μ_{m_2}	0.74	day^{-1}	u	$\left[0 \;\; 1.5\right]$	day^{-1}

Continuous stirred reactors are usually operated around a steady state. In order to make a good selection of the desired setpoint and to design an efficient control strategy to drive the system from an arbitrary initial state to the selected steady state, qualitative knowledge on the number and type of system equilibria is required along with conditions for their occurrence. Sbarciog et al. (2010) have shown that the set of equilibria of system (2)-(5) is globally convergent: as time increases every system solution converges to an equilibrium point, which all lie on the plane $\Delta = \left\{\xi \in \mathbb{R}^{+^4};\; \xi_1 + a\xi_3 = \xi_{in_1},\; \xi_2 - c\xi_3 + d\xi_4 = \xi_{in_2}\right\}$ Analytical expressions of the equilibria, as well as conditions for their occurrence in terms of the inputs u, ξ_{in_1}, ξ_{in_2} are provided. Here, we consider that the concentrations of the components in the influent ξ_{in_1} and ξ_{in_2} are fixed and known. Thus, the occurrence of the equilibria is determined entirely by the magnitude of the dilution rate u.

There are four types of steady states characterized by:

(1) **Total wash out**: there exists one steady state of this type, denoted by ξ_A, which is always physical, independent of the magnitude of the dilution rate;
(2) **Wash out of methanogenic bacteria**: there may exist only one steady state of this type, denoted by ξ_B, which occurs only if $u < \hat{u}$;
(3) **Wash out of acidogenic bacteria**: there may exist at most two steady states of this type, denoted by ξ_C and ξ_D, which occur simultaneously only if $u^* < u < \tilde{u}$, or only one, ξ_C, if $u \leq u^*$;
(4) **Coexistence of the two bacterial populations**: there may exist two steady states of this type, denoted by ξ_E and ξ_F, which occur simultaneously if $\bar{u} \leq u < \tilde{u}$, or only one, ξ_E, if $u < \bar{u}$. $\bar{u}$ is the dilution rate for which $\xi_{2,F} + c/a\xi_{1,F} = \xi_{in_2} + c/a\xi_{in_1}$, where $\xi_{1,F}$ is the solution of $\mu_1(\xi_1) = \bar{u}$ and $\xi_{2,F}$ is the higher solution of $\mu_2(\xi_2) = \bar{u}$.

Figure 1 shows the number and the type of equilibria for dilution rate values in each interval defined by the levels given in Table 2. These levels have been determined based on the system parameters given in Table 1.

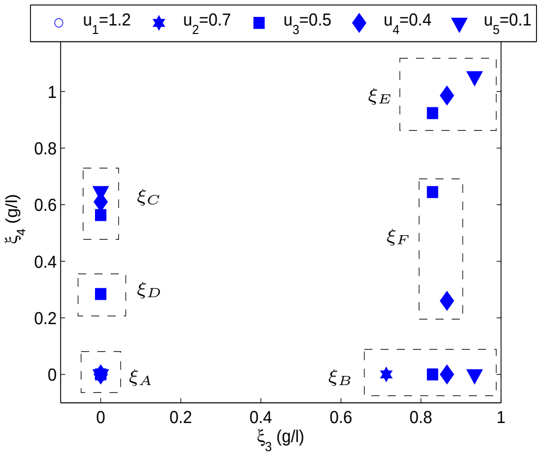

Fig. 1. Occurrence of equilibria for different values of the dilution rate

Table 2. Dilution rate levels and equilibria stability

Dilution rate	Equilibria (s - stable, u - unstable)
	ξ_A(s)
$\hat{u} = \mu_1(\xi_{in_1}) = 1.019$	
	ξ_A(u),ξ_B(s)
$\tilde{u} = \max(\mu_2(\xi_2)) = 0.536$	
	ξ_A(u),ξ_B(s),ξ_C(u),ξ_D(u),ξ_E(s),ξ_F(u)
$u^* = \mu_2(\xi_{in_2}) = 0.426$	
	ξ_A(u),ξ_B(s),ξ_C(u),ξ_E(s),ξ_F(u)
$\bar{u} = 0.3495$	
	ξ_A(u),ξ_B(u),ξ_C(u),ξ_E(s)

From a practical point of view, the only interesting equilibria are E and F, characterized by the presence of both bacterial populations, which ensure the conversion of the substrates and the production of biogas. Among these two steady states, higher outflow rate of biogas is obtained in the equilibrium E than in F (Sbarciog et al., 2012) as ξ_E is characterized by a higher concentration of methanogenic bacteria. Moreover, ξ_E is always stable while ξ_F is always unstable. Intuitively, high efficiency (in terms of the amount of treated water) of the anaerobic system and high biogas production are achieved by operating the process with high dilution rate such that the corresponding ξ_E is reached. However, ξ_E is the only stable equilibrium point of the system at low dilution rates ($u < \bar{u}$); for higher dilution rates ($u > \bar{u}$), ξ_B (methanogens wash out) and the operational point ξ_E are both locally asymptotically stable. Hence for some initial conditions, which may represent a wide subset of the state space, the system will be irreversibly driven to the acidification point ξ_B, where volatile fatty acids are accumulating in the reactor as their conversion to biogas does not take place due to the absence of methanogenic bacteria. Thus, an efficient control strategy is needed to safely start-up the reactor and drive it towards a meaningful steady state in terms of high biogas production and efficient treatment of wastewater.

3. CONTROL STRATEGY

In anaerobic digestion systems, the organic load is first converted to volatile fatty acids, which are subsequently consumed by the methanogenic bacteria to produce biogas. Thus a high production of biogas implies a high consumption of substrates, which is equivalent to an increased efficiency of the treatment process. Consequently, we state our control strategy from the perspective of driving the system towards a steady state where biogas production is maximum, as implicitly high consumption of substrates will be achieved. We evaluate the treatment efficiency in terms of the consumed COD (Chemical Oxygen Demand), which is typically around $60 - 70\%$ in anaerobic digestion systems. The COD consumption is calculated as

$$COD_{consumed}(\%) = \frac{COD_{in} - COD_{out}}{COD_{in}} \cdot 100 \qquad (9)$$

where the COD entering the reactor (COD_{in}) is computed assuming that ξ_{in_2} is mainly acetic acid,

$$COD_{in} = \xi_{in_1} + 0.06\xi_{in_2} \qquad (10)$$

while the COD leaving the reactor is given by the total organic matter in the effluent (organic matter is found also in the biomass)

$$COD_{out} = \xi_1 + 0.06\xi_2 + 1.42\xi_3 + 1.42\xi_4 \qquad (11)$$

To determine the equilibrium point ξ_s characterized by maximum outflow rate of biogas (further called the optimal setpoint) we perform a steady state optimization, which delivers also the dilution rate $u_s \in [u_{min}, u_{max}]$ corresponding to ξ_s. To this end, one uses the fact that the optimal setpoint ξ_s is an equilibrium point of type E, having the analytical expression (Sbarciog et al., 2010)

$$\xi_s = \begin{bmatrix} \xi_{1,s} \\ \xi_{2,s} \\ \dfrac{1}{a}\left(\xi_{in_1} - \xi_{1,s}\right) \\ \dfrac{1}{d}\left(\xi_{in_2} - \xi_{2,s} + c/a\left(\xi_{in_1} - \xi_{1,s}\right)\right) \end{bmatrix}$$

with $\xi_{1,s}$ the unique solution of

$$\mu_1(\xi_1) = u_s \qquad (12)$$

and $\xi_{2,s}$ the smaller solution of

$$\mu_2(\xi_2) = u_s \qquad (13)$$

Thus, in the optimal equilibrium point ξ_s the flow rate of methane is given by

$$Q(\xi_s) = q\mu_2(\xi_{2,s})\frac{1}{d}\left[\xi_{in_2} - \xi_{2,s} + \frac{c}{a}(\xi_{in_1} - \xi_{1,s})\right] \qquad (14)$$

Calculating $\left.\dfrac{dQ}{d\xi_2}\right|_{\xi_2=\xi_{2,s}} = 0$ leads to

$$\left[\xi_{in_2} - \xi_{2,s} + \frac{c}{a}(\xi_{in_1} - \xi_{1,s})\right] \cdot \mu_2'(\xi_{2,s}) - \mu_2(\xi_{2,s})\left(1 + \frac{c}{a}\left.\frac{d\xi_1}{d\xi_2}\right|_{\xi_2=\xi_{2,s}}\right) = 0 \qquad (15)$$

where $\left.\dfrac{d\xi_1}{d\xi_2}\right|_{\xi_2=\xi_{2,s}}$ results from

$$\mu_1'(\xi_{1,s})\left.\frac{d\xi_1}{d\xi_2}\right|_{\xi_2=\xi_{2,s}} = \mu_2'(\xi_{2,s}) \qquad (16)$$

with μ_2' denoting the derivative of μ_2 w.r.t. ξ_2 and μ_1' denoting the derivative of μ_1 w.r.t. ξ_1. Then (12), (13) and (15) provide sufficient conditions to fully determine the optimal equilibrium point ξ_s and the optimal dilution rate u_s.

Starting-up the operation of the reactor with the optimal dilution rate u_s is a risky choice. In steady state, the outflow rate of biogas $Q(\xi_s)$ equals the product $u_s \cdot \xi_{4,s}$ (as (13) holds). Since $Q(\xi_s)$ is maximum, then u_s assumes a high value. As emphasized in the previous section, at high dilution rates both the operational point and the acidification point are locally asymptotically stable. In this situation, the system convergence to an equilibrium or the other is determined by the system initial condition. Sbarciog et al. (2012) pointed out that as higher the dilution rate, as smaller the stability region of the operational point. Hence, unless starting-up the reactor in the neighborhood of the optimal setpoint, the operation is endangered of ending up in the acidification point. This is illustrated in Figure 2, which shows the system phase portrait and the boundary ($W^s(\xi_F^{u_s})$ - the stable manifold of the unstable equilibrium $\xi_F^{u_s}$) delimiting the stability regions of the optimal setpoint and of the acidification point on the Δ plane.

To overcome this problem we propose a simple switching strategy which moves the system from an arbitrary initial state to a small neighborhood S of the optimal setpoint ξ_s, further called the target set (Figure 2). Once the target set is reached, the dilution rate is changed to the optimal value u_s, such that the system settles down in the optimal setpoint. The transition between the initial state and a state lying inside the target set is carried out by measuring the outflow rate of biogas (the controlled variable) and by changing the dilution rate from u_{min} to u_{max} when a decrease in Q is noticed.

Recently, Sbarciog et al. (2012) have proposed a single switching strategy to approximate the solution of a free finite time optimal control problem, with the main goal of maximizing the outflow rate of biogas for the system (2)-(5) during the transient, while driving it towards the target set S. The structure of the control law is derived using the maximum principle of Pontryagin, however the switching

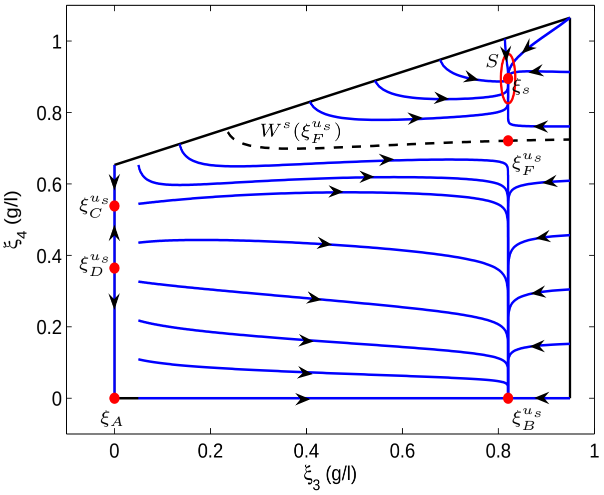

Fig. 2. The phase portrait on the Δ plane of the system operated with the dilution rate u_s

instant is chosen heuristically. This suboptimal control law relies on the model for estimating the system stability boundaries and on the knowledge of the full system state. The switching strategy proposed in this paper assumes the same structure but, although making use of the knowledge of system dynamics, requires only the measurement of the biogas outflow rate, which is one of the most common measurements in anaerobic digestion systems.

The switching control law is summarized as follows:

- operate the anaerobic digestion system with minimum dilution rate u_{min} until a decrease in the biogas outflow rate $Q(\xi)$ is detected;
- operate the system with maximum dilution rate u_{max} until the target set S is reached;
- operate the system with the optimal dilution rate u_s.

The success of this start-up procedure depends on one hand on the proper choice of u_{min}, u_{max} and the definition of S, and on the other hand on the switching instant. These topics are detailed below.

3.1 Definition of the control levels and of the target set

The control strategy relies on the system convergence to the set of equilibria, in particular to an equilibrium point of type E. Consequently, u_{min} and u_{max} must be chosen such that type E equilibria occur as physical equilibria for these levels of the dilution rate. This means (see Table 2)

$$u_{min} < u_{max} \leq \tilde{u} \tag{17}$$

One of the main purposes of the control strategy is to avoid the wash out of the methanogenic bacteria at start-up, which may occur due to an inappropriately high dilution rate for the system initial condition. Hence, in the first stage of the control interval, u_{min} must be low enough to prevent the occurrence of this phenomenon, at least from a large set of system initial conditions. Thus, one may choose

$$u_{min} \leq \bar{u} \tag{18}$$

In this case the system converges towards an equilibrium of type E from any initial condition characterized by the presence of both bacteria, as the acidification point is unstable (see Table 2). Alternatively, one may select

$$u_{min} > \bar{u} \tag{19}$$

but in this case, care must be taken for the system initial condition. As greater u_{min} is than $\bar{u}$, as larger the stability region of $\xi_B^{u_{min}}$ (the acidification point of the system operated with the dilution rate u_{min}) becomes.

The target set determines the moment of switching between u_{max} and u_s and it is defined as (Sbarciog et al., 2012):

$$\sum_{i=1}^{4} \frac{(\xi_i - \xi_{i,s})^2}{r_i^2} \leq 1 \tag{20}$$

The parameters r_i, $i = 1 \ldots 4$ must be chosen such that $\xi_E^{u_{max}}$ (the nominal equilibrium point of the system operated with the dilution rate u_{max}) lies in the target set S. This prevents the system to settle down in $\xi_E^{u_{max}}$, without the possibility to switch to the optimal dilution rate.

The minimum and maximum control levels have been selected as $u_{min} = 0.3\text{day}^{-1}$ and $u_{max} = 0.535\text{day}^{-1}$. The optimal dilution rate found from the steady state optimization is $u_s = 0.5179\text{day}^{-1}$ and the corresponding optimal setpoint is $\xi_s = [\,5.39\ 29.65\ 0.82\ 0.9\,]'$. The parameters of the target set have been chosen as $r_1 = r_2 = 0.01$, $r_3 = 0.015$, $r_4 = 0.07$.

3.2 Some considerations on the switching

Figure 3 illustrates the phase portrait on the Δ plane of the controlled system, in which one of the trajectories converges to the acidification point $\xi_B^{u_{max}}$. This occurs due to the fact that the controller switches between u_{min} and u_{max} at a moment when the system state is not yet in the region of attraction of $\xi_E^{u_{max}}$ (i.e. the system state lies below $W^s(\xi_F^{u_{max}})$). Note that, although the dilution rate is changed when a decrease in the biogas outflow rate is detected, this decrease may be either a small variation or may be related to a local maximum (as it is the case of the trajectory in Fig. 3). The evolution of the outflow rate of biogas for the system operated with u_{min} along the trajectory starting in the same initial state as the trajectory converging to the wash out of the methanogens is shown in Fig. 4. The switching instant is also indicated.

To overcome these situations, the switching strategy is reconsidered, such that switchings back (from u_{max} to u_{min}) are allowed whenever the biogas outflow rate decreases. This approach solves the problem of washing out the methanogens, but generates some undesired behaviour:

(1) high frequency switching for small variations in the biogas outflow rate: this can be avoided by allowing a small decrease in the biogas outflow rate before switching from u_{min} to u_{max} (further on a decrease of 0.5 from the highest value reached is allowed);
(2) unnecessary switching especially at high acidogens (ξ_3) and methanogens (ξ_4) concentrations: this can be avoided by allowing the controller to switch back from u_{max} to u_{min} only when a sufficient decrease in the biogas production has been noticed (further on a decrease of 15 from the highest value reached is allowed).

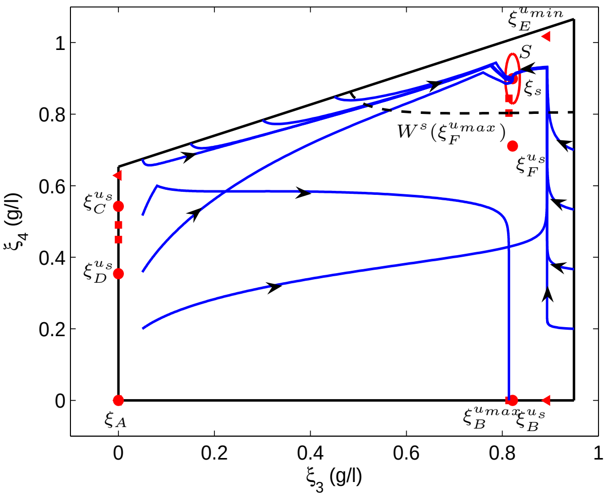

Fig. 3. The phase portrait on the Δ plane of the controlled system: one switching from u_{min} to u_{max}

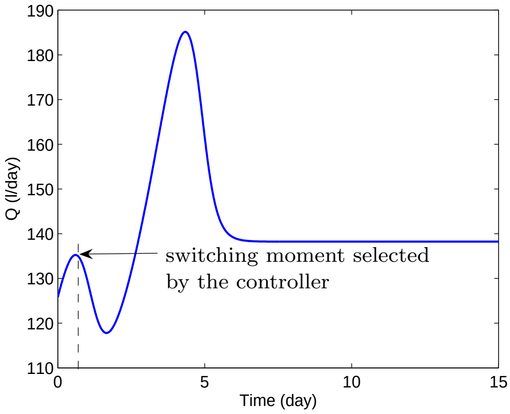

Fig. 4. The outflow rate of biogas for the system operated with u_{min} ($\xi(0) = [37.89\ \ 42.41\ \ 0.05\ \ 0.52]'$)

Figure 5 shows the response of the controlled system using the multiple switching approach for the initial state generating the trajectory converging to the wash out in the single switching approach.

4. COMPARISON WITH MINIMUM TIME OPTIMAL CONTROL

Comparing the results presented by Sbarciog et al. (2012) to the ones shown in Figure 5, one may notice that using the same levels for the dilution rate, the transition from the same initial condition to a state lying inside the target set is faster using the switching strategy proposed here. Hence, in the following we compare our results to the numerical solution of a minimum time optimal control problem.

The minimum time optimal control problem is formulated as: *Find the dilution rate* $u(t) \in [u_{min}, u_{max}]$ *which drives the system* (2)- (5) *from an initial state at time t=0 to the target set* S *given by* (20) *in minimum time.*

The numerical solution of this optimal control problem is found using the ACADO toolkit (Houska et al., 2011). Figure 6 shows the solutions obtained with the optimization software and the control law provided by the switching strategy for three system trajectories. The results are very similar, in the switching strategy the dilution rate is changed from u_{min} to u_{max} a bit later than in the optimal control strategy.

5. CONCLUSION

A simple switching strategy for an anaerobic digestion process has been presented in this paper, which drives the system from an arbitrary initial state to an optimal setpoint, characterized by maximum outflow rate of biogas. Additionally, high treatment efficiency is achieved, as in steady state more than 80% of the COD is removed. A fast convergence to the setpoint is obtained, the results being comparable to the numerical solution of the minimum time optimal control problem.

The switching strategy is developed based on the knowledge of system dynamics and can be applied to any anaerobic digestion process, for which the acidogenesis and methanogenesis are respectively characterized by Monod and Haldane kinetics, provided that the levels of the dilution rate are adjusted according to the system parameters. The main advantage of the method is that it requires only the measurement of the biogas outflow rate, which is easily obtained in anaerobic digestion processes. Although the target set is defined as a function of the system states in the present implementation for comparison reasons, also the switch from u_{max} to u_s can be made by only measuring the biogas. One possible implementation is to switch from u_{max} to u_s only after the system settled down in $\xi_E^{u_{max}}$ (then the biogas outflow rate is constant and known). Other definitions of the target set as a function of the biogas may be also considered.

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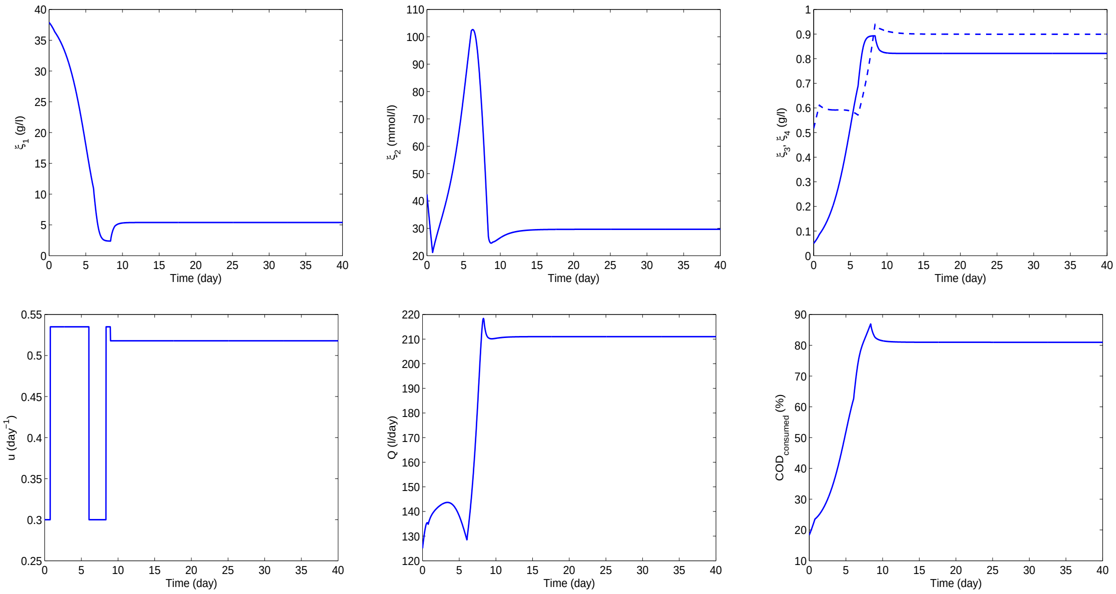

Fig. 5. Time evolution of the states, the control sequence, the outflow rate of biogas and the consumed COD for a system trajectory ($\xi(0) = [37.89 \;\; 42.41 \;\; 0.05 \;\; 0.52]'$)

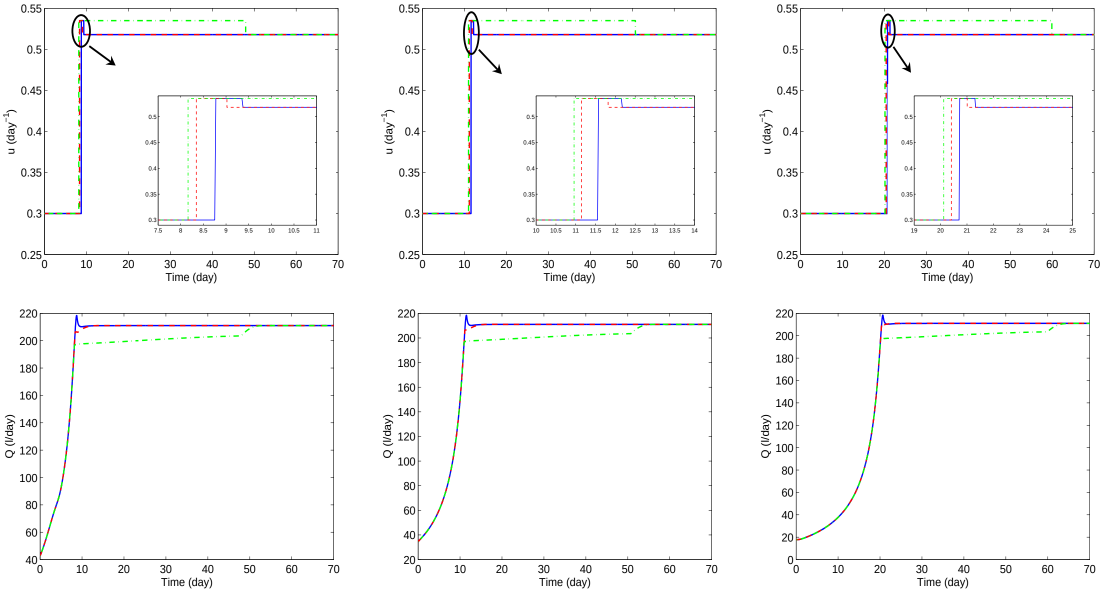

Fig. 6. Optimal control and switching control for three initial states: optimal control - dashed line, switching control - continuous line, suboptimal control as in Sbarciog et al. (2012) - dash-dotted line (S1: $\xi(0) = [37.89 \;\; 127.23 \;\; 0.05 \;\; 0.2]'$; S2: $\xi(0) = [0 \;\; 231.98 \;\; 0.95 \;\; 0.2]'$; S3: $\xi(0) = [14.716 \;\; 218.1 \;\; 0.6 \;\; 0.1]'$)

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